3D Subgrid Technique for the Finite Difference Method in the Frequency Domain

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Abstract— **A new, efficient finite difference frequency domain (FD-FD) subgrid technique is introduced. Based on a robust direct orthogonalization of the FD-FD grid, the method does not require any additional interpolation or correction terms. This yields a significant reduction in both CPU time and storage requirements as compared with the usual graded mesh techniques. The rigorous S-parameter calculation of 3D dielectric or metallic post element examples in rectangular waveguides demonstrates the versatility of the method. Comparisons with measurements and calculated reference values verify the presented technique and show its high dynamic range.**

I. Introduction

THE finite difference method [1] has found widespread application for the electromagnetic simulation of a great variety of waveguiding structures. Although its time domain (FD-TD) version has received higher popularity in recent years, the finite difference frequency domain (FD-FD) method has turned out to be a reliable technique for solving both complicated eigenvalue [2], [3], [4] and waveguide scattering problems [5], [6]. As usual microwave structures, such as obstacles in waveguides, often include regions of very different field intensity, the numerical effort both in CPU time and storage requirements can be high for accurate results if a uniform mesh is used. For rigorous and efficient simulations with the FD-FD method, therefore, an adequate subgrid technique is highly desirable.

Common graded mesh techniques [4], still lead to an unnecessarily fine mesh discretization in homogeneous areas of low field gradients, due to the topology of the grid. Locally refined meshes have been reported for the FD-TD technique in [7] - [10]. These FD-TD subgrid algorithms, however, require additional interpolation schemes at the grid-interfaces which can reduce the flexibility. Moreover, for the FD-FD method, no localized subgrid techniques are available so far.

The present paper introduces a FD-FD subgrid technique which utilizes the idea of a direct orthogonalization procedure successfully applied for the FD-TD method by the authors just recently [11]. The new technique does not require any additional interpolation or correction terms, and the refined mesh can be adapted most favorably to the structure where it is built up only in regions which need it directly. This makes the method very flexible, stable and efficient.

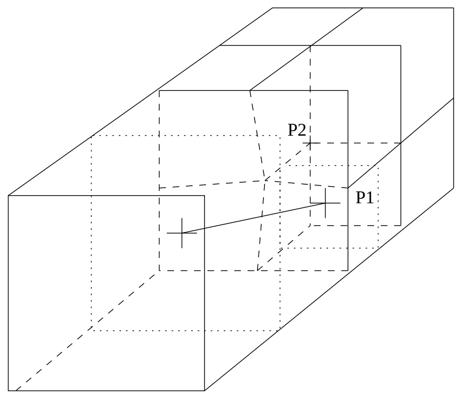

Fig. 1. Simple example for the 3D subgrid technique: Transition from a coarse grid to a subgrid within a waveguide. The contour lines of the subgrid are deformed such that the orthogonality with the magnetic field is performed.

II. Theory

Like in [11], first a recursive grid-generation procedure is used based on a progressive 2 : 1 cell ratio for the subgrids. From the allocated cells, two grids are derived (the main and the dual grid). The main grid is defined by the corners, the dual grid is defined by the centers of the allocated main cells. The main grid is directly orthogonalized against the dual grid [11]. An example for the three dimensional (3D) case is shown in Fig. 1, where the orthogonalization of the main grid is demonstrated.

The grids of the 3D subgrid discretization defines local basis-systems $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$, and the electric and the magnetic fields are represented by contra-variant components e_x^L and h_x^L, where x designates the corresponding component and L the grid level. An adequate condition forces each cell of level L to have neighbor cells in the levels $L-1$, L and $L+1$ only.

The electric field is given by

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$$e_{1}{}_{i,j,k}^{L_1} = \frac{1}{j\omega\epsilon A}\Big(h_{2}{}_{i,j,k-1}^{L_2}\ g_{22\,i,j,k-1} - h_{2}{}_{i,j,k}^{L_3}\ g_{22\,i,j,k} + h_{3}{}_{i,j,k}^{L_4}\ g_{33\,i,j,k} - h_{3}{}_{i,j-1,k}^{L_5}\ g_{33\,i,j-1,k}\Big)\,, \quad (1)$$

where $L_1 \geq L_2, L_3, L_4, L_5$, i, j, k denote the position of the grids in the mesh, and g_{ii} is the $i-$th diagonal element of the metric tensor with $g_{ii} = \vec{a}_i \cdot \vec{a}_i$; A is the area interspersed with the electric field. The magnetic field is obtained analogously.

This subgrid technique is stable, efficient and precise. Instead of using the dual grid of [11] (where the dual grid points are defined by the center points of the allocated main cells), a further improvement of the dynamic range is possible by an adequate shift of the dual grid points (e.g. P_1 in Fig. 1) towards the corners of the main grid (e.g. point P_2 in Fig.1). This enforces the orthogonality at a lover level of deformation.

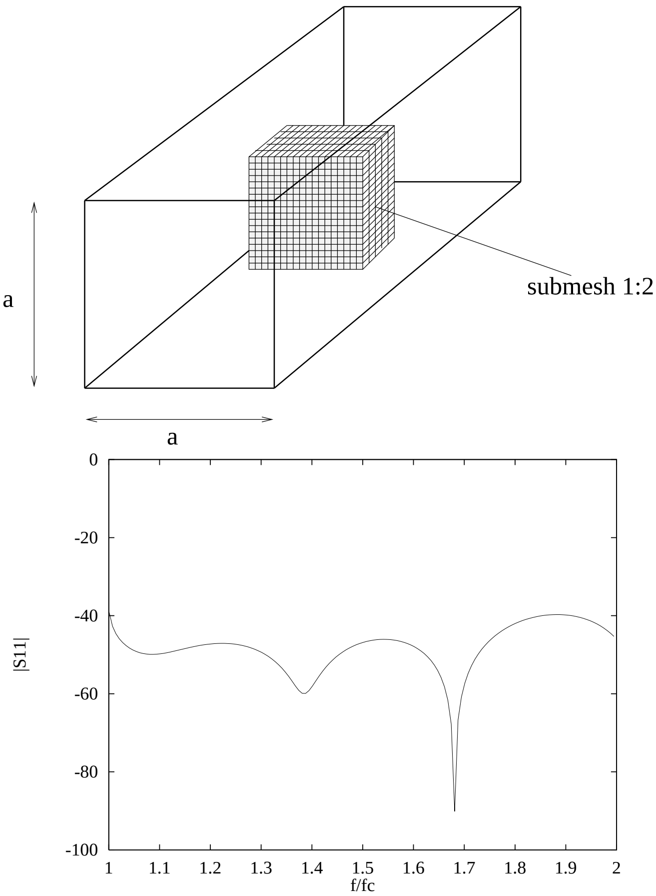

Fig. 2. Simple example of an empty homogeneous rectangular waveguide including a 3D subgrid region which does not border on the waveguide walls. The main grid (not shown) is discretized by 10×10 in transverse direction.
Return loss of the first two modes at the empty waveguide including the 3D subgrid region.

III. Results

The described 3D FD-FD subgrid technique has been tested first at the simple example of an empty homogeneous rectangular waveguide including a subgrid region (Fig. 2) which does not border on the waveguide walls. The main grid has been discretized in transversal direction by 10×10. For demonstrating the reliability of the presented subgrid technique, the return loss for the first two modes at the waveguide including the subgrid region has been calculated within the frequency range of $f_c \cdots 2 \times f_c$, where f_c denotes the corresponding cutoff frequencies. The return loss for both modes is the same and is better than -40 dB.

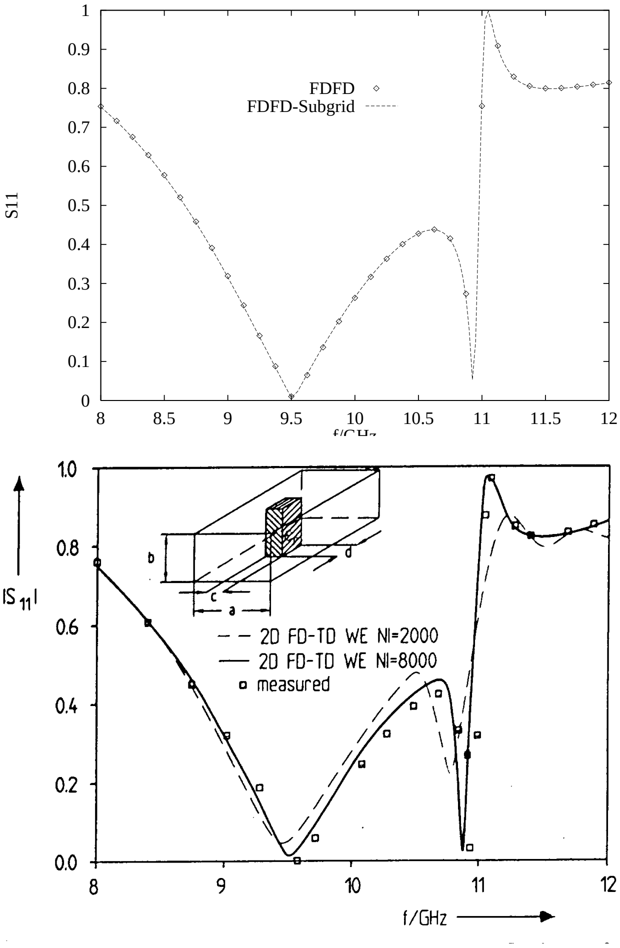

Fig. 3. Rectangular waveguide with dielectric insert. a=22.86 mm, b=10.16 mm, c=12 mm, d=6 mm, e_r=8.2. (a) Scattering parameter S_{11}, comparison between the FD-FD technique with a graded mesh (diamonds) and the FD-FD subgrid technique. Discretization in the cross section of the insert: 50 x 1 (graded mesh), 40 x 1 (subgrid). (b) Reference values by own calculations with the FD-TD technique and by measurements of [12].

Fig. 3a shows the results of a rectangular waveguide with a 3D dielectric insert for which reference measurements are available [12]. The chosen subgrid involves the inner structure of the dielectric block as well as its direct vicinity (Fig. 3b). The scattering parameter values S_{11} calculated with (dashed line) the subgrid and a graded mesh technique (diamonds) are plotted in Fig. 3a. Fig. 3b shows available measured results [12] and the comparison with the FD-TD technique [13] demonstrating the accuracy of the presented method.

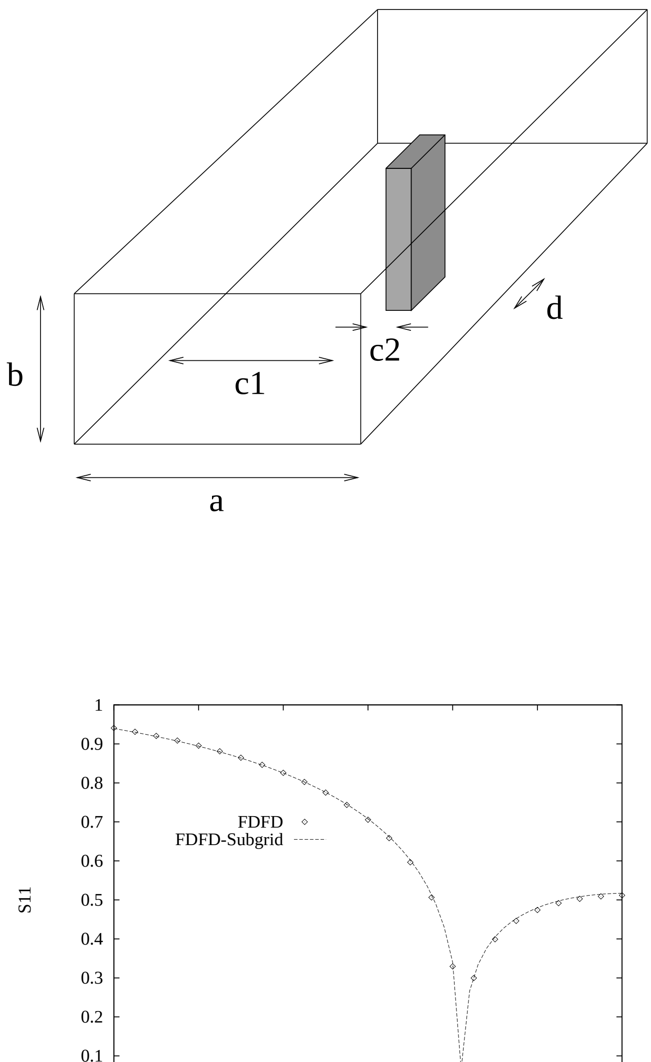

Fig. 4. Rectangular waveguide with a displaced metallic post: a=22.86 mm, b=10.16 mm, c1=12.86 mm, c2=1 mm, d=4 mm.
Comparison of the scattering parameter S_{11} calculated by the FD-FD subgrid technique with results obtained with a FD-FD graded mesh method (diamonds). Discretization in the cross section of the post: 50 x 1 (graded mesh), 40 x 1 (subgrid).

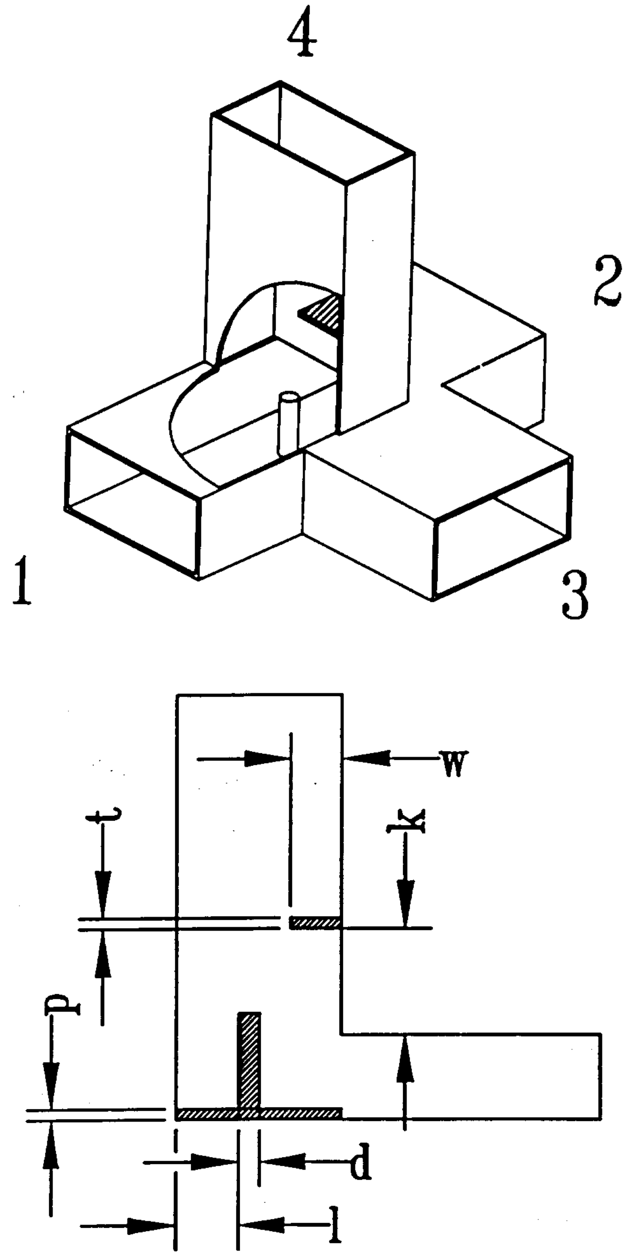

Fig. 5. Compensated magic tee structure. Dimensions in mm: Waveguide 12.7 x 25.4 , l=8.255, d=3.175, k=3.556, w=8.382, t=0.7937, p=0. Height of the post element: 12.7

The next example (Fig. 4) is a rectangular waveguide with a displaced rectangular metallic post. The scattering parameter S_{11} calculation shows again that identical results are obtained both for the FD-FD technique with a graded mesh (diamonds) and the FD-FD subgrid technique which leads to a reduced level of discretization.

Fig. 5 shows a compensated magic tee structure which has been calculated for demonstrating the applicability of the presented technique also for more complicated 3D cases. The post element and its immediate vicinity have been discretized by using a subgrid. Comparisons have been made with own calculations by a FD-TD technique where a uniform mesh has been applied and the post boundary has been discretized by a staircase approximation. It is demonstrated that - due to the high dynamic range and the finer discretization in the critical areas - the presented subgrid technique is more accurate and the resonant peaks in the return loss behavior are clearly detectable. This is particularly true for the S_{11} values (upper plot in Fig. 6) where the influence of the post element is more severe than for the S_{44}- and S_{41}-plots showing rather good agreement.

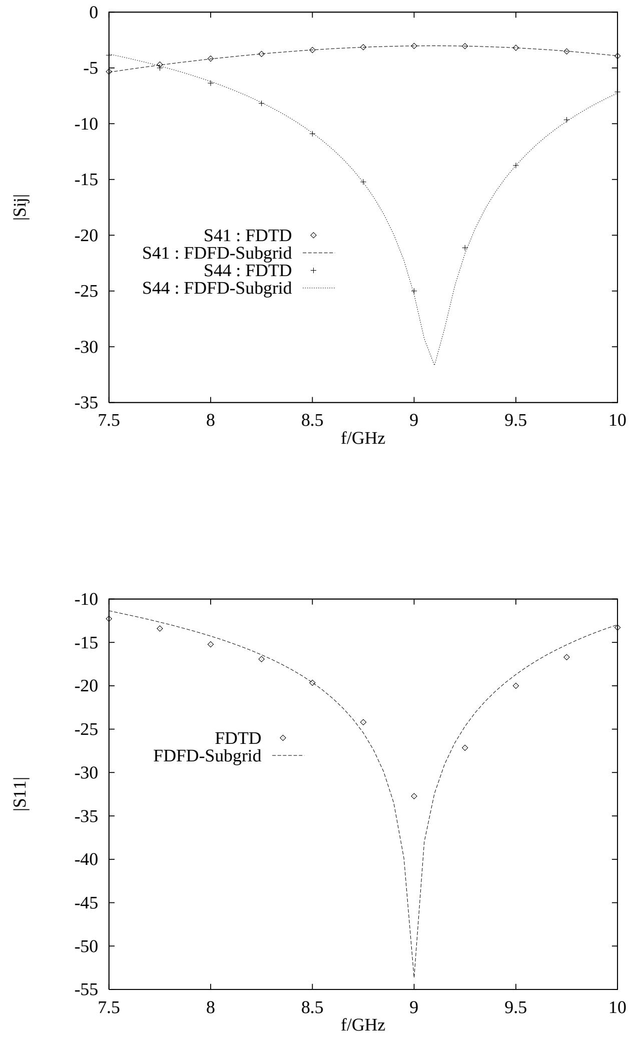

Fig. 6. Compensated magic tee structure of Fig. 5. Comparison of the scattering parameters S_{44}, S_{41}, S_{11} calculated by the FD-FD subgrid technique with results obtained by a FD-TD method (diamonds). Discretization for the subgrid technique: Ca. 400 cuboids in the waveguide cross-section, 968 cuboids with subgrid level 1 in the post and iris sections.

IV. Conclusion

A new, fast and stable FD-FD subgrid technique is described for the rigorous analysis of 3D microwave structures. The method does not require any additional interpolation or correction terms. This yields a significant reduction in both CPU time and storage requirements. Comparisons with measurements and calculated reference values verify the presented technique and show its high dynamic range.

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